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ON ISOTHERMAL GAS FLOW BETWEEN PARALLEL PLANES WITH ARBITRARY TANGENTIAL MOMENTUM ACCOMMODATION

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A method that is a modification of the method of half-space moments and uses the properties of collision integrals is proposed for the investigation of isothermal flow of a binary gas mixture between parallel planes. A formula is obtained for the slip velocity of a binary gas mixture between planes in the field of tangential pressure gradient with arbitrary accomodation of the tangential momentum. It is shown that the mixture isothermal slip is determined by viscous momentum transfer at the expense of the mean mass and diffusion velocities, and that the diffusion velocities tend to reduce the slip velocity by not more than 20%. It is noted that allowance for barodiffusion produces an additional term in the expression for the slip velocity of a binary mixture, whose order of magnitude is the same as that of diffusion slip velocity.

A considerable number of investigations (see, e.g., [1-7] was devoted to the isothermal slip of one-component gas in a field normal to the surface of mean velocity gradient. In more complex systems, such as binary gas mixtures, owing to the presence of relative motion due to the effect of partial concentration gradient, there exists one more form of isothermal slip of gas, viz. the diffusion slip of a binary mixture in the concentration gradient field normal to the surface [8, 9]. The investigation of gas slip in a binary mixture presents additional computation difficulties in comparison with a one-component gas. These difficulties made it impossible to extend the calculation of isothermal slip velocity, applicable to a Maxwell gas or one that interacts according to the law of solid sphere and based on the method of half-space expansions [2-5], to the case of binary mixture. Paper [10], where an attempt is made to investigate gas mixture slip for arbitrary molecule interaction, should be noted.

1. Let us consider the problem of Poiseuille flow of a binary gas mixture between two surfaces separated by a $2L$ distance. We set the system of coordinates between these with the X— and Y— axes, respectively, normal and parallel to the two surfaces. The distribution function for particles of the α kind is conveniently represented in the form

$$F_\alpha(\mathbf{r}, \mathbf{v}) = \frac{1}{2}(1 + \operatorname{sign} v_x) f_\alpha(\mathbf{r}, \mathbf{v}) + \frac{1}{2}(1 - \operatorname{sign} v_x) f_\alpha(\mathbf{r}, \mathbf{v}) \qquad (1.1)$$

where f_α and $f_{-\alpha}$ are distribution functions of impinging and reflected molecules, respectively, sign $v_x = 1$ in the half-space $v_x > 0$ and sign $v_x = -1$ in the half-space

$v_x < 0$, while function F_α is determined by the Boltzmann equation. We represent the distribution functions of impinging and reflected molecules in the form of finite series in Hermite-Tchebycheff polynomials [11]

$$f_{\pm\alpha}(\mathbf{r}, \mathbf{v}) = \frac{n_\alpha}{\pi^{3/2} v_{T_\alpha}^3} \exp\left\{-\frac{(v - u_{\pm\alpha y})^2}{v_{T_{\pm\alpha}}^2}\right\}\left\{1 - \right. \tag{1.2}$$

$$\left. \frac{\sigma_{\pm\alpha xy}}{2n_\alpha T_\alpha}\frac{m_\alpha}{T_\alpha}(v - u_{\pm\alpha})_y v_x - \frac{q_{\pm\alpha y}(v - u_{\pm\alpha})_y}{n_\alpha T_\alpha^2}\left[1 - \frac{m_\alpha}{5T_\alpha}(v - u_{\pm\alpha y})^2\right]\right\}$$

At distances from the surface greater than the mean free path, n_α, $u_{\pm\alpha y}$, $\sigma_{\pm\alpha xy}$, T_α, and $q_{\pm\alpha}$ denote, respectively, partial concentration, mean velocity, viscous stress tensor, temperature, and heat flux of particles of the α kind. It will be shown in the course of derivation of transport equations that the remaining expansion terms (e.g., $\sigma_{xx}u_x$, q_x, etc.) are quantities of a higher order of smallness with respect to parameter $M = um^{1/2}T^{-1/2}$.

One of the fundamental methods of investigation of gas motion dynamics is the method of moments. It involves the expansion of solution of the kinetic equation in series of orthogonal polynomials in the velocity space together with the Maxwell distribution function. The method was used in [12] for investigating gas slip along the surface. However the distribution function of the Knudsen layer has discontinuities of the first and higher derivatives in the velocity space, hence a better approximation of the distribution function is obtained by using the method of half-space moments in which the distribution function of impinging and reflected molecules are expanded in the half-space in orthogonal polynomials.

The derivation of transport equations by the method of moments stipulates multiplication of the Boltzmann equation by the velocity function $\psi_\alpha(v)$ followed by integration over the whole velocity space. Moreover, if one takes into account that the distribution function is represented by a finite series in orthogonal polynomials, this method reduces to the expansion of each terms of the Boltzmann equation in series in orthogonal polynomials with subsequent grouping terms at identical polynomials and, then, equating the obtained group to zero. At distances greater than the mean free path from the surface, the volume distribution function is best approximated by expanding each term of the kinetic equation in series in orthogonal polynomials throughout the space. The system of transport equations in the method of half-space moments is obtained in the same manner.

2. Let us pass to the integral of elastic collisions which we shall represent as the sum of two parts; $\Sigma_\beta\, G(F_\alpha, F_\beta)$ which is the part that determines the arrival of molecules in an element of the phase space, and $\Sigma_\beta D(F_\beta) F_\alpha$ which defines the departure of molecules from that element of the phase space. The quantities $G(F_\alpha(\mathbf{v}_\alpha'), F_\beta(\mathbf{v}_\beta))$ and $D(F_\beta(\mathbf{v}_\beta))$, where F_β is the distribution function of the Knudsen layer, are functions of v_α which have continuous derivatives of any order.

Functions $G(F_\alpha(\mathbf{v}_\alpha')F_\beta(\mathbf{v}_\beta'))$ and $D(F_\beta(\mathbf{v}_\beta'))$ are, actually, integral operators in which integration is carried out with respect to v_β and dissipation angles ϑ and φ. The dependence of molecule velocities after collisions on velocities prior to collision and on angles ϑ and φ is determined by the laws of conservation and may be conventionally represented in the form $\mathbf{v}_\alpha'(\mathbf{v}_\alpha, \mathbf{v}_\beta, \vartheta, \varphi)$ and $\mathbf{v}_\beta(\mathbf{v}_\alpha, \mathbf{v}_\beta, \vartheta, \varphi)$. Integrands in operators G and D and all their derivatives are continuous throughout the space of velocities $\mathbf{v}_\beta$, except the hypersurface (the manifold of points is of smaller dimension than the space of variables $\mathbf{v}_\beta, \vartheta$, and φ), where the Knudsen distribution function has discontinuous derivatives. For operator G these hypersurfaces are determined by the relationships $v_{\gamma x}'(\mathbf{v}_\alpha, \mathbf{v}_\beta, \vartheta, \varphi) = 0$, $\gamma = \alpha, \beta$, and for D by the relation $v_{\beta x} = 0$. Hence in conformity with the theory of integrals that depend on parameters [13] operators G and D and their derivatives of any order with respect to $\mathbf{v}_\alpha$ are continuous.

The elementary term appearing in the integral operator G is proportional to the elementary volume Δv_β in the vicinity of point $\mathbf{v}_\beta$. Because of this the input of terms with integrands discontinuous with respect to $\mathbf{v}_\alpha$ to the integral sum of the operator is small, since the hypersurface volume where derivatives of the integrand are discontinuous is infinitely small relative to the whole phase space. From the physical point of view the continuity of the first and higher derivatives with respect to $\mathbf{v}_\alpha$ on operator G is, thus, fairly evident.

Thus in conditions of the Knudsen layer all terms of the kinetic equation, except part G of the collision integral which determines the arrival of molecules in an element of the phase space, have discontinuous derivatives, and are, therefore, best approximated by the method of the half-space moment method. However, since part G derivatives are continuous, it can be approximated by conventional moments.

We use the latter for deriving the system of moment equations for the distribution function (1.2) represented in the form of expansion in Hermite-Tchebycheff polynomials orthogonal throughout the space.

One of the fundamental assumptions of the method of moments is the possibility of approximating the distribution function and each term of the kinetic equation by a finite series in orthoganal polynomials. The singularity of expansion (1.2) is that, unlike expansions by the method of conventional and half-space moments, it has in each of the velocity half-spaces it own expansion, as in the method of half-space moments, but these expansions are represented by series in polynomials that are orthogonal throughout the space, as in the method of conventional moments.

When deriving transport equations for the coefficients of (1.2) it is necessary to expand each terms of the kinetic equation in the half-spaces $v_{\alpha x} > 0$ and $v_{\alpha x} < 0$ in series in Hermite-Tchebycheff polynomials orthogonal throughout the space. For this the expansion (1.2) determined in one of the half-spaces is formally considered in the whole space. To obtain the coefficients of expansion (1.2) it is, then, sufficient to multiply the left-hand side of (1.2) by the corresponding Hermite-Tchebycheff polynomial and integrate it over the whole space.

Let us consider the terms of the kinetic equation (except G) whose derivatives are discontinuous at point $v_{\alpha x} = 0$ in one of the half-spaces. These terms have as the cofactor the distribution function (1.2) determined in that half-space. As previously, we consider the distribution function in the whole space. Then the final expansion of each term of the kinetic equation (except G) which, according to the assumptions of the method of half-space moments, exists in one of the half-spaces, is formally determined in the whole space of velocities v_α. The last statement follows from the linear independence of Hermite-Tchebycheff polynomials in each of the half-spaces. Thus in each of the half-spaces the coefficients of expansions of terms of the kinetic equation in series in Hermite-Tchebycheff polynomials are obtained by substituting for the complete distribution function F_α with discontinuous derivatives at $v_{\alpha x} = 0$, the distribution function $f_{\pm\alpha}$ for the appropriate half-space formally considered in the whole space. We then multiply the related term of the kinetic equation by polynomial $\psi_\alpha(\mathbf{v}_\alpha)$ and integrate it over the whole space.

As previously indicated, the collision integral $G(F_\alpha, F_\beta)$ which defines the arrival of molecules in a phase space element, has continuous derivatives at point $v_{\alpha x} \equiv 0$. To determine the coefficients of the expansion it is sufficient to multiply G by the Hermite-Tchebycheff polynomial and integrate over the whole space, and, then, when deriving transport equations consider the expansion separately in each of the half-spaces.

Applying the procedure of transport equation derivation described above, we obtain for the particular case of the distribution function (1.2) the following system of transport equations:

$$-\nabla_y n_\alpha T + \nabla_x \sigma^+_{\alpha xy} = \frac{n_\alpha n_\beta T}{nD_{\alpha\beta}}(u_\alpha - u_\beta) + \tag{2.1}$$

$$\sum_\beta \left(\frac{g_{\alpha\beta}}{\lambda^-_\beta} + \delta_{\alpha\beta}\frac{g_\alpha}{\lambda^-_\alpha}\right)\sigma^-_{\beta xy}$$

$$\nabla_x u^+_{\alpha y} = \sum_\beta \frac{a_{\alpha\beta}}{y_\alpha y_\beta}\sigma^+_{\beta xy} + \sum_\beta \left(\frac{h_{\alpha\beta}}{\lambda^-_\beta} - \delta_{\alpha\beta}\frac{h_\alpha}{\lambda^-_\alpha}\right)u^-_{\beta y}$$

$$\nabla_x \sigma^-_{\alpha xy} = \frac{\mu^-_\alpha}{(\lambda^-_\alpha)^2}u^-_{\alpha y}, \quad \nabla_x u^-_{\alpha y} = \frac{1}{\mu^-_\alpha}\sigma^-_{\alpha xy}, \quad T_\alpha = T_\beta = T$$

$$u^+_{\alpha y} = {}^1/_2(u'_{\alpha y} \pm u_{-\alpha y}), \quad \sigma^\pm_{\alpha xy} = {}^1/_2(\sigma_{\alpha xy} \pm \sigma_{-\alpha xy})$$

$$g_\alpha = 4\frac{\lambda^-_\alpha}{v_{T\alpha}}\sum_\beta\{M_\alpha^{3/2}\omega_{\alpha\beta}[t_y^2 t_x] + M_\alpha^{1/2}M_\beta\omega_{\alpha\beta} \times$$

$$[t_x w_y w'_y] - M_\alpha M_\beta^{1/2}\omega_{\alpha\beta}[t_y^2 w_x] - M_\beta^{3/2}\omega_{\alpha\beta}[w_x w_y \dot{w}'_y]$$

$$g_{\alpha\beta} = 4\,\frac{\lambda^-_\beta}{v_{T\beta}}\frac{n_\alpha}{n_\beta}M_\alpha\{M_\beta^{1/2}\omega_{\alpha\beta}[t_x w_y(w_y - w'_y)] +$$

$$M_\alpha^{1/2}\omega_{\alpha\beta}[w_x w_y(w_y - w'_y)]\}$$

$$h_{\alpha\beta} = 4\,\frac{\lambda^-_\beta}{v_{T\beta}}M_\alpha^{1/2}M_\beta^{1/2}\{M_\alpha^{1/2}\omega_{\alpha\beta}[t_x w_y(w_y - w'_y)]\}$$

$$h_\alpha = 4\frac{\lambda^-_\alpha}{v_{T\alpha}}\sum_\beta\{-M_\alpha^{1/2}\omega_{\alpha\beta}[t_x w_y w'_y] + M_\beta^{1/2}\omega_{\alpha\beta}[w_y(w_x w_y - w'_x w'_y)]\}$$

where $\omega_{\alpha\beta}[\varphi]$ represents a functional defined by formula

$$w_{\alpha\beta}[\varphi(\mathbf{t}, \mathbf{w}, \mathbf{w}')] = \frac{n_\beta}{(\pi v_{T\alpha}v_{T\beta})^3}\iint d\mathbf{t}d\mathbf{w}d\Omega\sigma_{\alpha\beta}(w, \vartheta) \times$$

$$w \operatorname{sign} v_{\alpha x}\exp\left\{-\frac{t^2}{v_0^2} - \frac{w^2}{v^2}\right\}\varphi\left(\frac{\mathbf{t}}{v_0}, \frac{\mathbf{w}}{v}, \frac{\mathbf{w}'}{v}\right)$$

$$\mathbf{v}_{\alpha x} = \mathbf{t}_x - M_\beta \mathbf{w}_x, \quad v_0{}^2 = v^2_{T\alpha}v^2_{T\beta}\,/\,v^2, \quad v^2 = v^2_{T\alpha} + v^2_{T\beta}$$

$$v_{T\alpha}{}^2 = 2T\,/\,m_\alpha, \quad M_\alpha = m_\alpha\,/\,(m_\alpha + m_\beta)$$

where $\sigma_{\alpha\beta}(w, \vartheta)$ is the differential cross section of diffusion, and $d\Omega$ is an element of the solid angle. Formulas for $a_{\alpha\beta}\lambda_\alpha^-$, and μ_α^- are given in [9].

The integral of elastic collisions retains the quantity, momentum, and energy of molecules. Hence the expansion of the collision integral in Hermite-Tchebycheff polynomials by the conventional method of moments has zero coefficients at polynomials of the zero, the three first and second order. The condition of conservation of the quantity of molecules, of over-all momentum and energy generally requires that, in the case of expansion of collision integrals by the method of half-space moments, the infinite series in coefficients at polynomials vanish. The condition of conservation by the collision integral of the y-component of momentum for the finite expansion (1.2) is defined by formula

$$\Big(\sum_{\beta} g_{\beta\alpha} + g_{\alpha}\Big) = \frac{2}{\sqrt{\pi}} (\mu_{\alpha}\ n_{\alpha} T)^{-1} \tag{2.2}$$

where g_{α}, $g_{\beta\alpha}$, and μ_{α}^{-} depend on the law of interaction between molecules consequently (2.2) is satisfied with varying accuracy for different laws.

A similar situation occurs, for instance, when the Chapman-Enskog method is used for solving the first approximation equation. In that case the distribution function is represented by a finite series and the expansion of the collision integral is limited to a finite series in polynomials. However the latter yields only a finite accuracy. Thus, for Maxwell molecules and finite expansion of the distribution function the collision integral is of the form of finite series in polynomials. In the case of an arbitrary law of collisions the expansion is also limited to a finite series. The first neglected term is then small: for small solid balls it is equal 1/20.

For Maxwellian molecules formula (2.2) is identically satisfied. In the case of arbitrary interactions it is necessary to take into consideration an infinite series if the condition of conservation of the y-component of the over-all momentum is to be satisfied. If, however, the expansion of (2.2) is limited to a finite series, the first neglected term is small. As noted above, for small solid balls that term is equal 1/20. Hence in order to avoid exceeding the exactness of solution of the Boltzmann equation by the Chapman-Enskog method it is advisable to limit the expansion of (1.2) to a finite series.

The laws of conservation of the number of particles, and of the x - and z - components of momentum and energy for the finite expansion of the collision integral are automatically satisfied, owing to the antisymmetry of the first and second order polynomials in (1.2) with respect to v_y.

3. Let us now turn to the slip velocity of a binary mixture of gas along a solid surface in the case of a Poiseuille flow. For the law of partial reflection of molecules from the surface we obtain from (2.1) for the slip velocity of a binary mixture the following formula

$$u = -\left\{\frac{\delta_1 a}{\delta_1 + \Delta} \frac{D_{\alpha\beta}\mu}{pL} [a_p]_1 + kw\mu\right\} \frac{\nabla p}{\mu} L \tag{3.1}$$

$$k = 1 - \frac{Da}{(\delta_1 + \Delta)\mu} (w_{\alpha}^{-1}\mu_{\beta} - w_{\beta}^{-1}\mu_{\alpha})$$

$$w^{-1} = \sum_{\beta} w_{\beta}^{-1}, \quad w_{\alpha}^{-1} = \sum_{\alpha} w_{\alpha\beta}^{-1}, \quad \delta_1 = \left(1 + \exp\left\{-\frac{2L}{\lambda}\right\}\right) \times \left(1 - \exp\left\{-\frac{2L}{\lambda}\right\}\right)^{-1}$$

$$\alpha = -\,[w_\alpha^{-1}\rho_\beta - w_\beta^{-1}\rho_\alpha]\,\rho^{-1}, \quad \Delta = Dw\,(w_{\alpha\alpha}^{-1}w_{\beta\beta}^{-1} - w_{\alpha\beta}^{-1}w_{\beta\alpha}^{-1})$$

$$w_{\alpha\alpha}^{-1} = a_{\alpha\alpha}d_{\alpha\alpha} - a_{\alpha\beta}d_{\beta\alpha}, \quad w_{\alpha\beta}^{-1} = a_{\alpha\beta}d_{\alpha\alpha} - a_{\alpha\alpha}d_{\alpha\beta}$$

$$a_{\alpha\alpha} = \frac{\mu_\alpha^- c_{\alpha\alpha}}{\Delta_d\lambda_\alpha^-}, \quad a_{\alpha\beta} = \frac{\mu_\beta^- c_{\alpha\beta}}{\Delta_d\lambda_\beta^-}, \quad D = \frac{nD_{\alpha\beta}}{n_\alpha n_\beta T\lambda}$$

$$\Delta_d = d_{\alpha\alpha}d_{\beta\beta} - d_{\alpha\beta}d_{\beta\alpha}, \quad c_{\alpha\alpha} = -\,(g_{\alpha\alpha} + g_\alpha)\,\delta_\alpha{}^+, \quad c_{\alpha\beta} = -\,g_{\alpha\beta}\delta_\beta{}^+$$

$$d_{\alpha\alpha} = \delta_{\varepsilon\alpha}^{-1}\left\{\left(1 + \frac{m_\alpha{}^\circ}{\sqrt{\pi}}\,\delta_{\varepsilon\alpha}\right) - \delta_{\varepsilon\alpha}\delta_\alpha{}^-\,(h_{\alpha\alpha} - h_\alpha)\right\}, \quad d_{\alpha\beta} = -\,h_{\alpha\beta}\delta_\alpha{}^-$$

$$m_\alpha{}^\circ = \frac{\mu_\alpha^-}{\lambda_\alpha^-}\,\frac{v_{T\alpha}}{n_\alpha T}, \quad \delta_{\varepsilon\alpha} = \frac{\varepsilon_\alpha}{2 - \varepsilon_\alpha}, \quad \delta_\alpha \pm = 1 \pm \exp\left\{-\,\frac{2L}{\lambda_\alpha^-}\right\}$$

where ε_α is the partial accommodation coefficient, $[a_p]_1$ is the coefficient of barodiffusion, and λ, λ_α, $D_{\alpha\beta}$, μ_α, and μ are given in [9].

In the case of a single gas with $L \gg \lambda$ the coefficients $[a_p]_1 \equiv 0$, $k = 1$, and (3.1) then becomes the formula for the slip of gas along a surface [14]. The dependence of the coefficient of a single gas isothermal slip on the accommodation coefficient is then the same as the coefficient determined in [15]. For a binary gas mixture the coefficient k in formula (3.1) is not unity, owing to the presence in it of a second term. That term takes into account the transport of viscous momentum by the diffusion velocity across the Knudsen layer, and vanishes for a single gas. At considerable distances from the surface coefficient k in the case of a Maxwellian binary mixture is represented by the formula

$$k = 1 + \frac{\delta_\varepsilon}{\sqrt{2}}\,\frac{(y_\alpha y_\beta)^{3/2}}{d\sqrt{\pi}}\left\{\frac{2}{(2-\varepsilon)\,d}\,\frac{\varepsilon_\alpha - \varepsilon_\beta}{\varepsilon_\alpha + \varepsilon_\beta} + \right.$$

$$\left. + \left(1 + \frac{\delta_\varepsilon}{d\sqrt{\pi}}\right)x_\alpha + \frac{4}{3}z_\alpha\right\}\left\{\left(1 - \frac{\delta_\varepsilon}{2\sqrt{\pi}d}\right)x_\alpha - \frac{3}{4}z_\alpha\right\}$$

$$x_\alpha = \frac{m_\alpha - m_\beta}{m_\alpha + m_\beta}, \quad z_\alpha = 2\,\frac{\sigma_\alpha - \sigma_\beta}{\sigma_\alpha + \sigma_\beta}$$

$$|\,\varepsilon_\alpha - \varepsilon_\beta\,| \ll (\varepsilon_\alpha + \varepsilon_\beta)\,/\,2 = \varepsilon, \quad d_{\alpha\alpha} = d_{\beta\beta} = d$$

Substituting this expression for k into (3.1), we find that the input to slip velocity due to the transport of viscous momentum across the Knudsen layer by diffusion velocities is defined by a quadratic form relative to x_α, z_α $(\varepsilon_\alpha - \varepsilon_\beta)$. Thus the slip coefficient is reduced by 20%, owing to diffusion transfer when $m_\alpha \gg m_\beta$, $\sigma_\alpha \gg \sigma_\beta$, $\varepsilon_\alpha = \varepsilon_\beta = 1$, and $y_\alpha = y_\beta = 1/2$.

Appearance of the first term in formula (3.1) for slip is due to the presence of barodiffusion in the system. Its magnitude is of the same order as the diffusion slip which occurs in systems with concentration gradients [8, 9]. The effect of the first term in formula (3.1) is smaller than that of the second term in proportion to $\mathrm{Kn} = \lambda\,/\,L$.

Note that at considerable distances from the surface outside the Knudsen layer u_α and $\sigma_{\alpha xy}$ tend to zero and the system of transport equations (2.1) becomes the system of Navier-Stokes equations for a mixture of gases, and the Knudsen distribution function becomes the Chapman-Enskog function.

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